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How experiment decides the $l=0, l=2$ split for the deuteron

Assume the D is a mixture of $L=0(s) \neq L=2(d)$ states

amplitudes

$$\vec{q}_D = a_s \vec{q}_s + a_d \vec{q}_d \quad (1)$$

$$1 = |\vec{q}_D|^2 = |a_s|^2 |q_s|^2 + |a_d|^2 |q_d|^2 = |a_s|^2 + |a_d|^2$$

Let $|a_s|^2 = f_s$ "fraction in s" (2)

$f_d = 1 - f_s =$ " " d

$$\langle \vec{q}_D | \mu | \vec{q}_D \rangle = f \langle \mu_s \rangle + (1-f) \langle \mu_d \rangle$$

$$\rightarrow \mu_m = f [\langle \mu_s \rangle - \langle \mu_d \rangle] + \langle \mu_d \rangle$$

measured
value for
the deuteron

So, $f = \frac{\mu_m - \langle \mu_d \rangle}{\langle \mu_s \rangle - \langle \mu_d \rangle}$ (3)

(4) $\begin{cases} \mu_s = \frac{\mu_N}{2} (g_{sn} + g_{sp}) & \text{from (11.4)} \\ = 0.87980463(5) & \text{in units of } \mu_N \end{cases}$

$$\mu_m = 0.857438(1)$$

$$\mu_D = \frac{1}{I+1} \langle I, m_I = I | \vec{\mu} \cdot \vec{I} | I, m_I = I \rangle$$

this is how it is measured.

Note

$\langle m_I = I \rangle$ is consistent, establishing the norm

So $\mu_D = \langle \vec{\mu} \cdot \vec{I} \rangle$

$$= \frac{1}{(I+1)} \left\langle \left(\frac{1}{2} g_{\mu p} \vec{L} + \frac{1}{2} \vec{S} (g_{sp} + g_{sn}) \right) \cdot \vec{I} \right\rangle$$

proton & neutron share

$\vec{L}, \vec{S}$ equally

$g_{sn} \equiv 0$ treated as

a "point" and it is neutral

$$\textcircled{5} \mu_D = \left[\frac{1}{2} g_{\mu p} \langle \vec{L} \cdot \vec{I} \rangle + \frac{1}{2} (g_{sp} + g_{sn}) \langle \vec{S} \cdot \vec{I} \rangle \right] / I+1$$

$\equiv 1$, by definition

$$\mu_D = \frac{1}{2(I+1)} \left[\langle \vec{L} \cdot \vec{I} \rangle + g_{sp} + g_{sn} \langle \vec{S} \cdot \vec{I} \rangle \right]$$

$$|\vec{I}|^2 - 2\vec{I} \cdot \vec{L} + |\vec{L}|^2 = |\vec{I} - \vec{L}|^2 = |\vec{S}|^2$$

$$\therefore \textcircled{6} \langle \vec{I} \cdot \vec{L} \rangle = \frac{1}{2} (I(I+1) + L(L+1) - S(S+1))$$

Similarly

$$|\vec{I}|^2 - 2\vec{S} \cdot \vec{I} + |\vec{S}|^2 = |\vec{I} - \vec{S}|^2 = |\vec{L}|^2$$

$$\text{so } \langle \vec{S} \cdot \vec{I} \rangle = I(I+1) + S(S+1) - L(L+1) \quad (7)$$

So, generally

$$\mu_d = \frac{1}{4(I+1)} \left\{ [I(I+1) + L(L+1) - S(S+1)] + (g_{sp} + g_{sn}) [I(I+1) + S(S+1) - L(L+1)] \right\}$$

Hence $I=1, L=0, S=1$

$$\mu_d = \frac{1}{2} (g_{sp} + g_{sn})$$

$I=1, L=2, S=1$

$$\mu_d = \frac{1}{8} \{ 6 + (g_{sp} + g_{sn})(-2) \}$$

$$= [3 - (g_{sp} + g_{sn})]/4$$

Put this into (3)

$$f = \frac{\mu_m - [3 - (g_{sp} + g_{sn})]/4}{\frac{1}{2}(g_{sp} + g_{sn}) - [3 - (g_{sp} + g_{sn})]/4}$$

$$= \frac{\mu_m - \frac{3}{4} + \frac{1}{4}(g_{sp} + g_{sn})}{\frac{3}{4}(g_{sp} + g_{sn}) - \frac{3}{4}}$$

(4)

$$f = \frac{\mu_m - \frac{1}{2}(q_{sp} + q_{sn}) + \frac{3}{4}(-3 + (q_{sn} + q_{sp}))}{\frac{3}{4}((q_{sn} + q_{sp}) - 3)}$$

$$f = 1 - \frac{\mu_m - \frac{1}{2}(q_{sp} + q_{sn})}{\frac{3}{4}(3 - (q_{sn} + q_{sp}))}$$